Math 112 Quiz 14 Pledged Key
20 points

Quiz grade________________ Sum of points____________

1. Derive a power series expression with c = 0 for f(x) = Arctan(x) and then show the series that would represent $\frac{-\pi}{4}$.

$$f'(x) = \frac{1}{1+x^2} = \frac{1}{1-(-x^2)} \Rightarrow \sum_{n=0}^{\infty} (-1)^n x^{2n}$$

$$f(x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{2n+1} + C$$

series

$$f(0) = \text{Arctan}(0) = 0 \Rightarrow C = 0$$

$C = 0$

$$f(-1) = \text{Arctan}(-1) = -\frac{\pi}{4}$$

$f(-1) = \frac{-\pi}{4}$

$$f(-1) = \sum_{n=0}^{\infty} \frac{(-1)^n (-1)^{2n+1}}{2n+1}$$

$$f(-1) = \sum_{n=0}^{\infty} \frac{(-1)^{3n+1}}{2n+1} = \frac{-\pi}{4}$$

2. Derive the McLauren power series for f(x) = sin(x) and then find a series for $g(x) = \dfrac{\sin(4x^2)}{x^3}, x \neq 0$. Give the domain.

$$f(x) = \sin x \qquad f(0) = 0$$
$$f'(x) = \cos x \qquad f'(0) = 1$$
$$f''(x) = -\sin x \qquad f''(0) = 0$$
$$f'''(x) = -\cos x \qquad f'''(0) = -1$$
$$f^{(4)}(x) = \sin x \qquad f^{(4)}(0) = 0$$

$$f(x) = x - \frac{1}{3!}x^3 + \frac{1}{5!}x^5 - \frac{1}{7!}x^7 + \frac{1}{9!}x^9 \ldots$$
$$= \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!} = \sin x$$

$$g(x) = \frac{\sin(4x^2)}{x^3} = x^{-3} f(4x^2)$$
$$g(x) = x^{-3} \sum_{n=0}^{\infty} \frac{(-1)^n (4x^2)^{2n+1}}{(2n+1)!} = \boxed{\sum_{n=0}^{\infty} \frac{(-1)^n 4^{2n+1} x^{4n-1}}{(2n+1)!}}$$

Domain:

$$\lim_{n\to\infty} \left| \frac{4^2 \cdot \cancel{4^{2n+1}} x^4 \cdot \cancel{x^{4n-1}}}{(2n+3)(2n+2)\cancel{(2n+1)!}} \cdot \frac{\cancel{(2n+1)!}}{\cancel{4^{2n+1}} \cancel{x^{4n-1}}} \right| < 1$$

$$|2^4 x^4| \lim_{n\to\infty} \frac{1}{(2n+3)(2n+2)} \to 0 < 1$$

Domain: all $\mathbb{R}$