Math 107 (110 points) Test 3 Pledged Key

1. True/False. If the statement is true, indicate TRUE in the blank to the left; if the statement is false, indicate FALSE in the blank to the left AND correct the statement so that the statement is true (30 points).

True 1). A point estimate is a numerical value that is the estimate of a parameter.

True 2). The larger the number in a sample, the smaller the confidence interval as long as the rest of the information remains constant.

True 3). As n gets really large (goes to infinity), the student t distribution becomes the standard normal distribution.

False 4). The F-distribution is not symmetric ~~just~~ but as the standard normal and the student t distributions are symmetric.

True 5). Both the F-distribution and t-distribution vary with degrees of freedom.

True 6). Both the student t distribution and the standard normal distribution have means of zero.

False 7). The null hypothesis for the claim that the length of fish is at least 36 inches is $H_o: \cancel{\mu \le 36}$. $\mu \geq 36$ [Claim is null for this.]

True 8). The rejection region is in the right tail for the hypothesis $H_o: \mu \le 36$.

True 9). A type II error occurs if the null is false but one does not reject the null hypothesis.

True 10). The level of significance is given by the Greek letter alpha and generally is below .05 if the null hypothesis is rejected.

True 11). A two-tailed statistical test of the difference of two means (with n = 35 in one sample and n = 92 in another sample) is rejected with p < .05, with a test statistic value of - 2.01. ±1.96 C.V.

False 12). For a test statistic of z = 2.51 in a two-tailed test, the p-value is ~~.006~~. .012

False 13). We are generally unable to determine β since we do not know the population ~~mean~~ variance for the distribution or distributions being compared.

False 14). When the value of alpha is increased, the probability of committing an error is ~~decreased~~. increased

True 15). If a 95% confidence interval for the difference of two proportions contains zero, then the related null hypothesis should not be rejected.

Part II: Problems:

1. The noise levels at urban hospitals were measured in decibels. The mean of such noise levels in 84 corridors was 61.2 decibels and the standard deviation was 7.9.

a) Construct a 95% confidence interval and interpret the meaning. (10)

$n = 84 \quad \bar{x} = 61.2 \quad s = 7.9 \quad \alpha = .05$

$\bar{x} \pm E$ where $E = z_{\frac{\alpha}{2}} \frac{s}{\sqrt{n}} = 1.96\left(\frac{7.9}{\sqrt{84}}\right)$

$E = 1.689$

$61.2 \pm 1.7 \Rightarrow 59.5 < \mu < 62.9$

95% confident that the true population decibel level is inside the interval.

b) It was believed that the noise level is 60 decibels in urban hospitals. <u>Without testing hypotheses, using the information from your confidence interval</u>, state what the null hypothesis is and the expected conclusion. (10)

$H_0: \mu = 60$

We would fail to reject $\mu = 60$, the null hypothesis, since 60 is in the interval of $59.5 < \mu < 62.9$.

2. (10 points) A researcher is interested in estimating the average SAT score in a large urban school district. She wants to be 98% confident that her estimate is correct. If previous studies showed that the standard deviation is 108 points and the expected return rate of the survey is 85%, how large a sample is needed to be accurate within 30 points?

$S = 108$ $\quad 98\% \rightarrow \alpha = .02, \ \frac{\alpha}{2} = .01$

$E = 30$

$z_{\frac{\alpha}{2}} = 2.326$

$$n = \left(\frac{z_{\frac{\alpha}{2}} S}{E}\right)^2 = \left[\frac{(2.326)(108)}{30}\right]^2 \doteq 70.1$$

$$n = \boxed{71}$$

Now 85% return rate:

$.85x = 71$

$x = \frac{71}{.85} \doteq 83.5$ so $\boxed{84}$

3. The average price of 15 cans of beans is \$1.26 with a standard deviation of \$.055. The average price of 24 cans of peas is \$1.30 with a standard deviation of \$.038.

a) (10 points) Homogeneity of variances was checked, giving test statistic $F = 2.09$. What conclusion was reached and why?

C.V. $F(14, 23)$ not on chart

$F(12, 23) = 2.57$; $F(15, 23) = 2.47$

$F = 2.09$ not in rejection region

Fail to reject $H_0: \sigma_1^2 = \sigma_2^2$

So data meet the assumption of homogeneity of variances.

b) (10 points) Is there a significant difference in the price of beans and peas? The test statistic value is –2.689. State the null hypothesis and your conclusion using an appropriate alpha level and explain your choice.

Null Hypothesis $H_0: \mu_1 = \mu_2$ or $\mu_1 - \mu_2 = 0$

Conclusion (give alpha level):

$t(37) \rightarrow$ use $Z = 2.576$, $\alpha = .01$

$t = -2.689$ is in rejection region.

Reject $\mu_1 = \mu_2$ at $\alpha = .01$

There is a significant difference between the price of cans of peas and cans of beans.

4. (15 points) In an effort to improve the vocabulary of 10 students, a teacher provides a weekly 1-hour tutoring session for them. A test is given before the sessions, and another test is given afterward. The results are shown in the table. At $\alpha = 0.01$, can the teacher conclude that the tutoring sessions helped to improve the students' vocabularies? Assuming that there are no outliers and the set of data, the difference, is not significantly skewed.

Student	1	2	3	4	5	6	7	8	9	10
After-Tutoring	88	82	100	72	84	75	79	71	81	70
Before-Tutoring	83	76	92	64	82	68	70	68	72	63
Difference	5	6	8	8	2	7	9	3	9	7

Null hypothesis $\mu_D \leq 0$

$\bar{D} = 6.4$

$s \doteq 2.4129$

$n = 10$

$$t = \frac{\bar{D} - \mu_D}{s_D/\sqrt{n}} = \frac{6.4 - 0}{2.4129/\sqrt{10}}$$

$t = 8.387$

Test statistic 8.387

Critical Value $t(9) = 2.821$ $\alpha = .01$, one-tail

Conclusion and interpretation:

Reject the null hypothesis ($\mu_D \leq 0$)

The tutoring seems to make a difference, scores are significantly higher after tutoring.

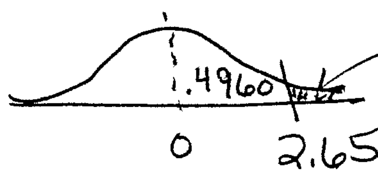
5. (15 points) A congressional candidate claims that a majority of voters think that the war with Iraq has made the U.S. less safe from terrorism. A poll of 1008 adults in the congressional district showed that 546 of them agree with the candidate. Is there sufficient evidence to support the candidate's claim?

Null Hypothesis $p \le \frac{1}{2}$ $\qquad$ $p > \frac{1}{2}$ claim

$$\hat{p} = \frac{546}{1008} \qquad Z = \frac{\hat{p} - p}{\sqrt{\frac{pq}{n}}} = \frac{.5417 - .5}{\sqrt{\frac{(.5)(.5)}{1008}}}$$

$$Z = 2.648$$

Test statistic 2.65



p-value is this area

p-value $p = .004$ $\qquad$ $.5 - .4960$

Conclusion and interpretation:

Reject the null hypothesis ($p \le \frac{1}{2}$) at $p = .004$.

The congressional candidate seems correct in that a significant majority believe that the war with Iraq has made the US less safe from terrorism.