Math 112 Quiz 11 Pledged Key

Test for convergence. State the test used and your conclusions clearly. If appropriate, determine if the convergence is absolute or conditional: (9 points each0

1) $\sum_{n=1}^{\infty}(-1)^n \sin\left(\frac{1}{n}\right)$ ① By AST, $0 < \sin\left(\frac{1}{n+1}\right) < \sin\left(\frac{1}{n}\right)$ and $\lim_{n\to\infty} \sin\left(\frac{1}{n}\right) = \sin(0) = 0$, therefore the alternating series converges. ② Consider $\sum_{n=1}^{\infty} \sin\left(\frac{1}{n}\right)$: LCT using $\sum \frac{1}{n}$, p-series, $p=1$, diverges, as follows $\lim_{n\to\infty} \frac{\sin\left(\frac{1}{n}\right)}{\frac{1}{n}}$ "$\frac{0}{0}$" $\overset{L'H}{=} \lim_{n\to\infty} \frac{\cos\left(\frac{1}{n}\right)\left(\cancel{-n^{-2}}\right)}{\cancel{-n^{-2}}}$

$= \cos(0) = 1 > 0$ and finite, therefore $\sum_{n=1}^{\infty} \sin\left(\frac{1}{n}\right)$ diverges. ③ So, $\sum_{n=1}^{\infty}(-1)^n \sin\left(\frac{1}{n}\right)$ converges conditionally.

2) $\sum_{n=1}^{\infty} \frac{3^n n!}{(n+1)^n}$

$a_{n+1} = \frac{3 \cdot 3^n (n+1) n!}{(n+2)^{n+1}}$

By ratio test,

$$\lim_{n\to\infty} \left| \frac{3 \cdot \cancel{3^n}(n+1)\cancel{n!}}{(n+2)(n+2)^n} \cdot \frac{(n+1)^n}{\cancel{3^n}\cancel{n!}} \right| = 3 \lim_{n\to\infty} \left(\frac{n+1}{n+2}\right)^{1} \left(\frac{n+1}{n+2}\right)^{n \;\to\; \frac{1}{e}}$$

$\lim_{n\to\infty} \left(\frac{n+1}{n+2}\right)^n$ "1^∞" Let $y = \left(\frac{n+1}{n+2}\right)^n$

$\ln y = n \ln\left(\frac{n+1}{n+2}\right)$ $\quad \cancel{n}+2-\cancel{n}-1$

$$\lim_{n\to\infty} \frac{\ln\left(\frac{n+1}{n+2}\right)}{\frac{1}{n}} \text{ "}\tfrac{0}{0}\text{" } \overset{L'H}{=} \lim_{n\to\infty} \frac{\left(\frac{n+2}{n+1}\right)\left(\frac{(n+2)-(n+1)}{(n+2)^2}\right)}{-\frac{1}{n^2}} \overset{Alg}{=}$$

$$\lim_{n\to\infty} \left(\frac{1}{(n+1)(n+2)}\right)\left(\frac{-n^2}{1}\right) = -1 \Rightarrow \text{so, } e^{-1} \text{ OR } \frac{1}{e}$$

$$3 \lim_{n\to\infty} \left(\frac{n+1}{n+2}\right)^{1} \left(\frac{n+1}{n+2}\right)^{n} = \frac{3}{e} > 1, \text{ diverges}$$