Math 111, Quiz 12 **PLEDGED**________________________

Setup, but do not evaluate, the integrals needed to find the volume formed by rotating the area bound between the graphs of the following equations about each of the indicated lines. For each line, do this in two ways, using the "washer method" and using the "cylindrical shell method". For your convenience, drawings of this region with the related axes of rotation have been provided on the second page.
(*22 pts total*)

intersection: $2y = -y^2 + 6y \rightarrow y^2 - 4y = 0 \rightarrow y(y-4) = 0$

$y=0 \quad y=4$

$(0,0) \quad (8,4)$

$x = -y^2 + 6y$ and $x = 2y$

$y = \frac{1}{2}x$

$y^2 - 6y + x = 0$

$y = \frac{6 \pm \sqrt{36 - 4(1)(x)}}{2} = \frac{6 \pm 2\sqrt{9-x}}{2}$

$y = \frac{\cancel{2}(3 \pm \sqrt{9-x})}{\cancel{2}} = 3 \pm \sqrt{9-x}$

$x = -y^2 + 6y$

$x' = -2y + 6$

$y = 3$

$(9,3)$

vertex

1. Rotated about the line $y = 6$,

(washer method) $\pi\int_0^8 \left[(6-(3-\sqrt{9-x}))^2 - (6-\tfrac{1}{2}x)^2\right]dx + \pi\int_8^9 \left[(6-(3-\sqrt{9-x}))^2 - (6-(3+\sqrt{9-x}))^2\right]dx$

(cylindrical shell method) $2\pi\int_0^4 (6-y)(-y^2+6y-2y)\,dy$

2. Rotated about the line $x = 10$,

(washer method) $\pi\int_0^4 \left[(10-2y)^2 - (10-(-y^2+6y))^2\right]dy$

(cylindrical shell method) $2\pi\int_0^8 (10-x)\left[\tfrac{1}{2}x - (3-\sqrt{9-x})\right]dx$

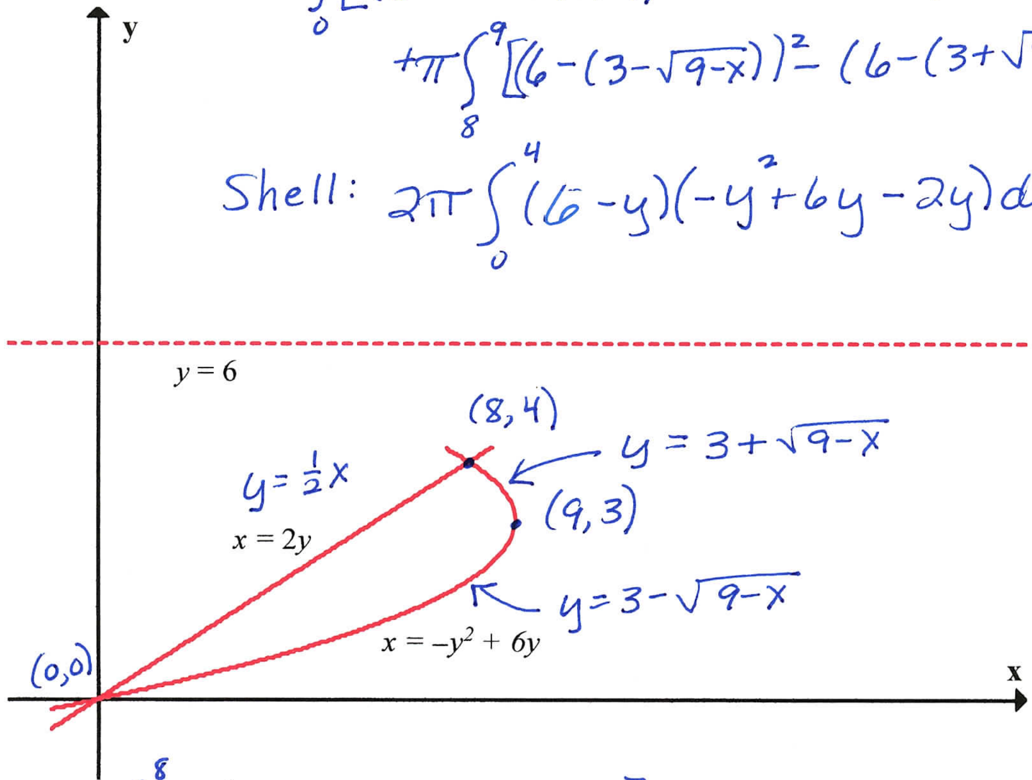
$2\pi\int_8^9 (10-y)\left[(3+\sqrt{9-x}) - (3-\sqrt{9-x})\right]dx$

Disc: $\pi\int R_o^2 - R_I^2$

$$\pi\int_0^8 \left[(6-(3-\sqrt{9-x}))^2 - (6-\tfrac{1}{2}x)^2\right]dx$$

$$+\pi\int_8^9 \left[(6-(3-\sqrt{9-x}))^2 - (6-(3+\sqrt{9-x}))^2\right]dx$$

Shell: $2\pi\int_0^4 (6-y)(-y^2+6y-2y)\,dy$



Shell: $2\pi\int_0^8 (10-x)\left[\tfrac{1}{2}x-(3-\sqrt{9-x})\right]dx$

$$+2\pi\int_8^9 (10-x)\left[(3+\sqrt{9-x})-(3-\sqrt{9-x})\right]dx$$

Disc: $\pi\int_0^4 \left[(10-2y)^2-(10-(-y^2+6y))^2\right]dy$