Math 107 (110 points) Test 2 Pledged Key

1. (11 points) Given the following:

x	2	3	4
$P(x)$	1/2	1/3	1/6

a) Explain clearly why the above is a pdf. (5 points)

1.) $\sum P(x) = \frac{1}{2} + \frac{1}{3} + \frac{1}{6} = 1$

2.) $0 \le \frac{1}{6}, \frac{1}{3}, \frac{1}{2} \le 1$

b) Find the mean and variance (6 points)

$\mu = \sum x \cdot P(x) = 2\cdot\frac{1}{2} + 3\cdot\frac{1}{3} + 4\cdot\frac{1}{6} = 2\frac{2}{3} \doteq 2.67$

$\mu = \frac{8}{3} \doteq 2.667$

Aside: $\sum x^2 \cdot P(x)$ $4\cdot\frac{1}{2} + 9\cdot\frac{1}{3} + 16\cdot\frac{1}{6} = 2+3+\frac{8}{3} = \frac{23}{3}$

$\sigma^2 = \sum x^2 \cdot P(x) - \mu^2 = \frac{23}{3} - \left(\frac{8}{3}\right)^2 = \frac{23}{3} - \frac{64}{9}$

$= \frac{5}{9}$

$\sigma^2 = \frac{5}{9} \doteq .556$

Formulas:

$\mu = \sum xP(x)$

$\sigma^2 = \sum[x^2P(x)] - \mu^2$

$z = \frac{x-\mu}{\sigma}$

$z = \frac{\bar{x}-\mu}{\sigma/\sqrt{n}}$

$P(x) = {}_nC_x p^x q^{n-x}$ ($\mu = np$ $\sigma = \sqrt{npq}$)

$P(x;\lambda) = \frac{e^{-\lambda}\lambda^x}{x!}$

$P(x) = \frac{n!}{x_1! \cdot x_2! \cdots x_k!} \cdot p_1^{x_1} \cdot p_2^{x_2} \cdots p_k^{x_k}$

2. (35 points) In a neighborhood 25% of the households have tents. If eighteen households are selected at random and let the random variable x be the number of households have tents, respond to the following:

$p = .25 \quad q = .75 \quad n = 18$

a) (5 points) Find the mean and standard deviation of this distribution.

$\mu = np = 18(.25) = 4.5$

$\sigma = \sqrt{npq} = \sqrt{18(.25)(.75)} \doteq 1.8371$

b) (5 points) What is the probability that exactly three have tents?

$P(3) = {}_{18}C_3 (.25)^3 (.75)^{15}$

$P(3) \doteq .17$

c) (10 points) What is the probability that at least three have tents?

$$1 - P(X<3)$$

$$1 - [P(0) + P(1) + P(2)]$$

$$1 - [{}_{18}C_0(.25)^0(.75)^{18} + {}_{18}C_1(.25)^1(.75)^{17} + {}_{18}C_2(.25)^2(.75)^{16}]$$

$$\doteq 1 - [.00564 + .03383 + .09584]$$

$$1 - .13531 \doteq .8647$$

d) (10 points) Approximate the probability in part b) by using the standard normal.

$$P(2.5 < x < 3.5)$$

$$Z = \frac{x-\mu}{\sigma} : \quad Z = \frac{2.5-4.5}{1.8371} \doteq -1.09$$

$$Z = \frac{3.5-4.5}{1.8371} \doteq -.54$$

$$P(-1.09 < x < -.54) = .3621 - .2054$$

$$= .1567$$

e) (5 points) Was your approximation appropriate? Explain clearly why.

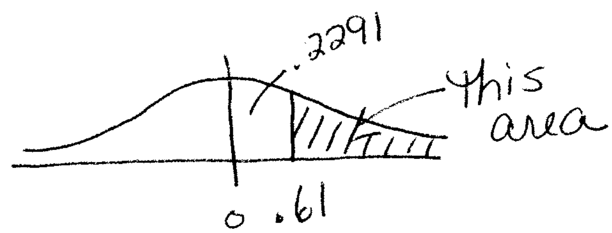
No, $np = 4.5$ so the condition requiring $np \geq 5$ AND $nq \geq 5$ does not hold.

3. (40 points) A company claims that their pain pills contain 200 mg of ibuprofen. Therefore, the mean is assumed to be 200 mg and, past quality control measures gives the standard deviation as 16.4 mg and an approximate normal distribution of weights. Respond to the following: $\mu = 200mg \quad \sigma = 16.4$

   a) What is the probability that a randomly selected pill will have more than 210 mg of ibuprofen?

$$P(X > 210) \doteq P(Z > .61) \doteq .5 - .2291$$

$$Z = \frac{X-\mu}{\sigma} = \frac{210-200}{16.4} \doteq .61 \qquad = \boxed{.2709}$$

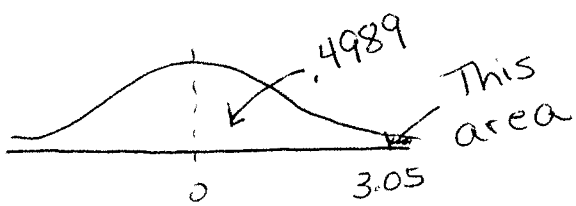


   b) What is the probability that the mean of a sample of 25 pills will have more than 210 mg of ibuprofen?

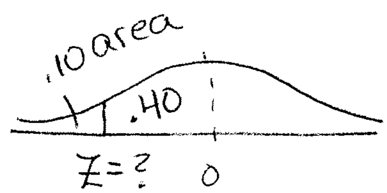
CLT

$$P(\bar{X} > 210) = P(Z > 3.05) \doteq .5 - .4989$$

$$Z = \frac{\bar{X}-\mu}{\sigma/\sqrt{n}} = \frac{210-200}{\frac{16.4}{\sqrt{25}}} \doteq 3.05 \qquad \doteq \boxed{.0011}$$



c) What is the weight for the bottom 10% cut-off?



Look up $Z = -1.28$

Solve for x:

$$Z = \frac{X - \mu}{\sigma} \rightarrow -1.28 = \frac{x - 200}{16.4}$$

$$x = (-1.28)(16.4) + 200$$

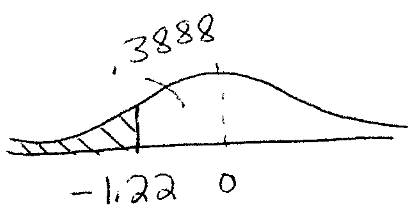
$$x \doteq 179 \text{ mg}$$

d) How many pills in a bottle of 250 would you expect to have less than 180 mg of ibuprofen in them?

$$250 \cdot P(x < 180) = 250(.1112) \Rightarrow$$ about 28 pills

$$P(x < 180) = P(Z < -1.22) = .5 - .3888 = .1112$$

$$Z = \frac{X - \mu}{\sigma} = \frac{180 - 200}{16.4} = -1.22$$



4. (12 points) Typos occur on an average of one every 80 pages of print. In a book of 400 pages, what is the probability that at most four pages will have typos?

Poisson

$\lambda = \frac{1}{80} \cdot 400 = 5$ average for 400 pages

$P(0) + P(1) + P(2) + P(3) + P(4)$

where $P(x) = \frac{e^{-5} 5^{x}}{x!}$, as follows:

$$\frac{e^{-5}5^0}{0!} + \frac{e^{-5}5^1}{1!} + \frac{e^{-5}5^2}{2!} + \frac{e^{-5}5^3}{3!} + \frac{e^{-5}5^4}{4!}$$

$$e^{-5}\left(\frac{5^0}{0!} + \frac{5}{1!} + \frac{5^2}{2!} + \frac{5^3}{3!} + \frac{5^4}{4!}\right) = e^{-5}\left(1 + 5 + \frac{25}{2} + \frac{125}{6} + \frac{625}{24}\right)$$

$$\doteq .44$$

5. (12 points) A drug store special on a case of cokes usually produces a fast sale. A single customer can not buy more than four cases. The probability that a customer will buy none, one, two, three, four cases of cokes is 0.45, 0.15, 0.2, 0.12, 0.08. For the next 10 customers, what is the probability that three will not buy any cokes, two will buy one case each, three will buy two cases each, one will buy three cases and one will buy four cases?

Multinomial

buy	prob	x
0	.45	3
1	.15	2
2	.2	3
3	.12	1
4	.08	1

$$\frac{10!}{3!\,2!\,3!\,1!\,1!}(.45)^3(.15)^2(.2)^3(.12)(.08)$$

$$\doteq \boxed{.0079}$$